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Modeling and multi-objective optimization of powder mixed electric discharge machining process of aluminum/alumina metal matrix composite

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ABSTRACT

Low material removal rate (MRR) and high surface roughness values hinder large-scale application of electro discharge machining (EDM) in the fields like automobile, aerospace and medical industry. In recent years, however, EDM has gained more significance in these industries as the usage of difficult-to-machine materials including metal matrix composites (MMCs) increased. In the present work, an attempt has been made to fabricate and machine aluminum/alumina MMC using EDM by adding aluminum powder in kerosene dielectric. Results showed an increase in MRR and decrease in surface roughness (R_a) compared to those for conventional EDM. Semi empirical models for MRR and R_a based on machining parameters and important thermo physical properties were established using a hybrid approach of dimensional and regression analysis. A multi response optimization was also performed using principal component analysis-based grey technique (Grey-PCA) to determine optimum settings of process parameters for maximum MRR and minimum R_a within the experimental range. The recommended setting of process parameters for the proposed process has been found to be powder concentration (C_p) = 4 g/l, peak current (I_p) = 3 A, pulse on time (T_{on}) = 150 μs and duty cycle (T_{au}) = 85%.

1. Introduction

Electro discharge machining (EDM), one of the most popular non-conventional machining processes, is an electro-thermal process in which work piece is usually submerged in a liquid dielectric medium and shaped through the action of a succession of high frequency discrete electrical discharges (sparks) produced by a DC pulse generator. Every spark locally erodes (melts and vaporizes) tiny amount of the material surface, the overall effect being a cavity as the complementary shape of tool electrode geometry over the work surface. Besides tool and die making, in the recent years, EDM has found many applications in the fields of automobile, aerospace, surgical instruments making and military industry. The major problem to use EDM for large scale production is its low machining rate and poor surface finish. Many advances have come in the field of EDM to overcome these difficulties. One of those advancements is powder mixed EDM (PMEDM). Even though, the principle of PMEDM is not completely understood, the results of the experiments done by many researchers have shown significant improvement in material removal rate (MRR) and surface quality. Among the powder materials, aluminum produced better MRR and good surface finish compared to other materials like Cr, Cu and SiC [1,2]. This was attributed to the increased spark gap due to aluminum's high electrical conductivity and low density. Kerosene or some commercial EDM oil was widely used as the dielectric medium in PMEDM [1,2,4–16]. However, Yan et al. [3] established the feasibility of urea suspended water as a dielectric medium by achieving a smooth machined surface. MRR and surface quality can be further enhanced by providing ultrasonic vibration to the tool [4–6]. Apart from expelling the debris from the machine zone, the vibration provides the abrasive action on work-piece surface. Tzeng and Lee [2] investigated the effect of thermo-physical properties of work-piece material and various additives (Al, Cu, Cr, SiC) on the efficiency of EDM. Multi-objective optimization of PMEDM process was performed by Kansal et al. [7] used Taguchi technique to optimize

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the process parameters during machining of AISI D2 die steel using silicon suspended dielectric. Singh and Yeh [8] employed grey relational analysis (GRA) for the multi-objective optimization of high MRR and low R_a for PMEDM of 6061 Al/Al_2O_3 aluminum matrix composite (MMC) using SiC particles suspended dielectric. Kumar and Davim [9] studied the role of silicon powder suspended dielectric on the material removal mechanism during PMEDM of Al10%SiC_P MMC. Garg et al. [10], in their review on the EDM of MMCs, discussed the effectiveness of PMEDM for the machining of MMCs. Tsai and Wang [11] developed a semi empirical model for the surface finish for the conventional EDM process with the help of non-linear optimization technique. Patil and Brahmankar [12] developed a semi-empirical model for MRR using the thermo-physical properties of the MMC and compared it with the model generated using response surface methodology (RSM).

Very little research work has been found in the area of PMEDM of MMCs fabricated using stir casting method [8,9]. Owing to more homogeneous distribution of reinforcement in the matrix material along with minimum possibility of oxidation of MMCs, the current research work attempted to fabricate aluminum/alumina MMC using powder metallurgy technique. The machining characteristics of the fabricated MMCs were further investigated during PMEDM process. Models for MRR and R_a were developed using dimensional analysis in which thermo-physical properties of work-piece material were considered along with machining parameters. Furthermore, determination of optimal combination of process parameters was achieved through multi-objective optimization using PCA based grey relational analysis.

2. Experimental details

2.1. Fabrication of MMC

Powders of aluminum and alumina of sizes about 15 μm and 90 μm were taken in the weight ratio of 80:20 and were thoroughly milled in a high energy ball mill for 2 h at 300 rpm. Images of different powder particles captured using scanning electron microscopy (SEM) before and after ball milling operation are shown in Fig. 1. Liquid toluene was used as process control agent. Ball to powder ratio was kept to 10:1. MMC samples were fabricated using conventional compaction and sintering technique. Compaction was performed at pressure of 250 MPa. Acetone was used as the lubricant to prevent the agglomeration of the particles. The green compacted specimens were sintered in argon atmosphere at a temperature of 500 °C and allowed to cool to room temperature for 24 h. The sintered specimens were heated to 400 °C and quenched in iced water. They were further heated to 200 °C and allowed to cool in the muffle furnace for 8 h to avoid natural aging.

2.2. Machining of MMC

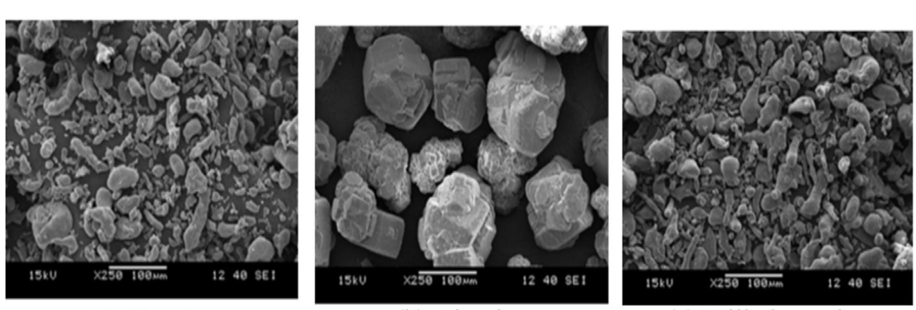
All experiments were performed using a die sinking EDM set up (Model: LEADER-1ZNC, ELECTRONICA). A special arrangement has been made to the EDM set up for proper circulation and mixing of additive particles in the dielectric. Aluminum powder particles (with an average size of 15 μm) were suspended in kerosene which was used as dielectric. Experiments were performed using Taguchi L18 orthogonal array. Copper electrode of diameter 12 mm was used to machine the Al/Al_2O_3 MMC to a depth of 2 mm. The effect of powder concentration (C_p), peak current (I_p), pulse on time (T_{on}) and duty cycle (T_{au}) on responses MRR and R_a was studied. Fig. 2 shows photographic view of the MMC specimens fabricated using the powders shown in Fig. 1, along with shallow blind holes made using powder-mixed EDM process.

2.3. Measurement of responses

Density of the MMC specimens was found to be 2.5 g/cc. Weight of each specimen before and after machining was measured using a high precision balance (Model: VIBRA, SANSUI Electronics). Machining time was recorded using a stop watch. Surface roughness in the form of R_a (center line average) was measured at three randomly chosen locations on each machined surface using a portable stylus type profilometer (Model: Talysurf, Surtronic 3+, Taylor Hobson) and the mean value was calculated.

3. Modeling using dimensional analysis

Dimensional analysis is a method of dimensions and a mathematical technique which deals with the physical quantities involved in the experiments to formulate a model for the response in terms of control parameters as well as some physical properties of the materials. This is based on the hypothesis that the solution of the problem is expressible by means of a dimensionally homogeneous equation. By using the hybrid approach of regression and dimensional analysis, authors tried to incorporate the material properties into the model equations which may not be possible using evolutionary optimization techniques. Buckingham π theorem states that it is possible to assemble all variables appearing in a problem into a number of dimensionless π terms [17,18]. The thermal properties, physical properties along with machining parameters might determine the performance of PMEDM process of MMCs. In the present model, the thermo-physical properties like thermal conductivity, density and coefficient of thermal expansion were considered along with the machining parameters such as peak current, pulse on time, powder concentration, duty cycle and average gap voltage. The dimensions of these variables as well as their values are mentioned in Table 1.



(a) Aluminum (b) Alumina (c) Milled powder

Fig. 1. SEM images of powder particles.

Fig. 2. Machined work-pieces.

3.1. Modeling of MRR

Dimensionless equation for MRR can be given as follows.

$$Q = f(C_p, I_p, T_{on}, \tau, V_g, K, \alpha, \rho) \quad (1)$$

Buckingham's π theorem is used to assemble all the variables appearing in the problem in a number of dimensionless products. It is necessary to choose repeating variables to develop a semi empirical model using Buckingham's π theorem. Five variables were selected as repeating variables on the basis of suitable conditions.

The equation must satisfy dimensional homogeneity to get a complete set of dimensionless products. In the present case, the fundamental dimensions are mass (M), length (L), time (T), current (I), temperature (θ). In case of MRR modeling, total variables (n) are 9 and fundamental dimensions (n) are 5 so that there will be 4 ($n - r$) π terms.

Therefore, functional relationship for π term can be expressed as,

$$g(\pi_1, \pi_2, \pi_3, \pi_4)$$

Using Buckingham's π theorem each dimensionless π terms are evaluated and shown below:

$$\pi_1 = \rho^{a1} I_p^{b1} K^{c1} \alpha^{d1} V_g^{e1} Q \quad (2)$$

$$\pi_2 = \rho^{a2} I_p^{b2} K^{c2} \alpha^{d2} V_g^{e2} T_{on} \quad (3)$$

$$\pi_3 = \rho^{a3} I_p^{b3} K^{c3} \alpha^{d3} V_g^{e3} C_p \quad (4)$$

Table 1
Details of machining parameters, material properties of MMC and output responses in PMEDM process.

	Factor	Symbol	Unit	Dimension
Output responses	Material removal rate	Q	mm^3/min	L^3T^{-1}
	Surface roughness	R_a	µm	L
Machining parameters	Pulse on time	T_{on}	µs	T
	Peak current	I_p	A	I
	Powder concentration	C_p	g/l	ML^{-3}
	Duty cycle	τ	µs/µs	$M^0L^0T^0I^0\theta^0$
	Gap voltage	V_g	V	$ML^2T^{-3}I^{-1}$
Material properties	Density	ρ	g/cm^3	ML^{-3}
	Thermal conductivity	K	Cal/mole°C	$MLT^{-3}\theta^{-1}$
	Coefficient of thermal expansion	α	°C^{-1}	θ^{-1}

$$\pi_4 = \rho^{a4} I_p^{b4} K^{c4} \alpha^{d4} V_g^{e4} \tau \quad (5)$$

After equating both sides dimensionally, powers of each parameter are obtained and rearranged to get dimensionless form as given below

$$\pi_1 = \frac{\rho^{\frac{1}{3}} K^{\frac{4}{3}} Q}{I_p^{\frac{5}{3}} \alpha^{\frac{4}{3}} V_g^{\frac{4}{3}}} \quad (6)$$

$$\pi_2 = \frac{K^{\frac{5}{3}} \alpha^{\frac{5}{3}} T_{on}}{I_p^{\frac{4}{3}} V_g^{\frac{4}{3}} \rho^{\frac{1}{3}}} \quad (7)$$

$$\pi_3 = \frac{C_p}{\rho} \quad (8)$$

$$\pi_4 = \tau \quad (9)$$

As π_1 is a function of other three π terms, hence we can write as:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4) \quad (10)$$

$$\frac{\rho^{\frac{1}{3}} K^{\frac{4}{3}} Q}{I_p^{\frac{5}{3}} \alpha^{\frac{4}{3}} V_g^{\frac{4}{3}}} = f\left(\frac{K^{\frac{5}{3}} \alpha^{\frac{5}{3}} T_{on}}{I_p^{\frac{4}{3}} V_g^{\frac{4}{3}} \rho^{\frac{1}{3}}}, \frac{C_p}{\rho}, \tau\right)$$

$$Q = A \frac{I_p^{\frac{5}{3}} \alpha^{\frac{4}{3}} V_g^{\frac{4}{3}}}{\rho^{\frac{1}{3}} K^{\frac{4}{3}}} \left(\frac{K^{\frac{5}{3}} \alpha^{\frac{5}{3}} T_{on}}{I_p^{\frac{4}{3}} V_g^{\frac{4}{3}} \rho^{\frac{1}{3}}}\right)^a \left(\frac{C_p}{\rho}\right)^b (\tau)^c \quad (11)$$

3.2. Modeling of R_a

Functional relationship of surface roughness can be expressed as,

$$R_a = f(C_p, I_p, T_{on}, \tau, V_g, K, \alpha, \rho) \quad (12)$$

Dimensional analysis is done here to relate the process parameters of PMEDM with surface roughness and each π term relations are shown below.

$$\pi_5 = \rho^{a1} I_p^{b1} K^{c1} \alpha^{d1} V_g^{e1} R_a \quad (13)$$

$$\pi_6 = \rho^{a2} I_p^{b2} K^{c2} \alpha^{d2} V_g^{e2} T_{on} \quad (14)$$

$$\pi_7 = \rho^{a3} I_p^{b3} K^{c3} \alpha^{d3} V_g^{e3} C_p \quad (15)$$

$$\pi_8 = \rho^{a4} I_p^{b4} K^{c4} \alpha^{d4} V_g^{e4} \tau \quad (16)$$

After equating both sides dimensionally & and rearranging equations, dimensionless terms obtained are given as below.

$$\pi_5 = \frac{KR_a}{\alpha V_g I_p} \tag{17}$$

$$\pi_6 = \frac{K^{\frac{5}{3}}\alpha^{\frac{5}{3}}T_{on}}{I_p^{\frac{4}{3}}V_g^{\frac{4}{3}}\rho^{\frac{1}{3}}} \tag{18}$$

$$\pi_7 = \frac{C_p}{\rho} \tag{19}$$

$$\pi_8 = \tau \tag{20}$$

As π_1 is the function of other three π terms, hence we can write as:

$$\pi_5 = f(\pi_6, \pi_7, \pi_8) \tag{21}$$

$$\frac{KR_a}{\alpha V_g I_p} = f\left(\frac{K^{\frac{5}{3}}\alpha^{\frac{5}{3}}T_{on}}{I_p^{\frac{4}{3}}V_g^{\frac{4}{3}}\rho^{\frac{1}{3}}}, \frac{C_p}{\rho}, \tau\right)$$

$$R_a = B\frac{\alpha V_g I_p}{K}\left(\frac{K^{\frac{5}{3}}\alpha^{\frac{5}{3}}T_{on}}{I_p^{\frac{4}{3}}V_g^{\frac{4}{3}}\rho^{\frac{1}{3}}}\right)^d \left(\frac{C_p}{\rho}\right)^e (\tau)^f \tag{22}$$

A multiple linear regression analysis is carried out to predict the values of a dependent variable Y, given a set of 'n' dependent variables (X_1,X_2,X_3,…,X_n)

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_n X_{ni} + \varepsilon_i \tag{23}$$

where Y_i is the response and β_0,β_1,β_2,β_3,…,β_n are regression parameters, X_0,X_1,X_2,X_3,…,X_n are covariates and ε_i is the error. It can be written in the matrix form as follows.

$$Y_{n+1} = X_{n*(r+1)}{}^*\beta_{(r+1)*1} + \varepsilon_i \tag{24}$$

$$\widehat{Y} = X\widehat{\beta} \tag{25}$$

$$\widehat{\beta} = \left(X^T X\right)^{-1} X^T Y \tag{26}$$

$$\widehat{\varepsilon} = Y - \widehat{Y} \tag{27}$$

where Y is the observed value and $\widehat{Y}$ is the predicted value for this experiment.

After regression analysis MRR and surface roughness equation are given below.

$$\text{MRR} = 2.4*10^{-35}\frac{C_p^{14.21} I_p^{14.07} V_g^{13.83} T_{on}^{9.37} \tau^{0.2}}{(K\alpha)^{14.38}\rho^{11.42}} \tag{28}$$

$$R_a = 7.5*10^{18}\frac{C_p^{2.78}(I_p V_g)^{4.84}}{(K\alpha)^{3.8} T_{on}^{2.88}\rho^{1.83}} \tag{29}$$

4. Optimization using Grey-PCA

It is important for any process to follow optimal combination of process parameters to achieve desired output response by utilizing minimum possible resources. The optimal parameter setting for a single response may be detrimental for other responses at the same time. So a compromise has to be made and a multi-objective optimization has to be performed to arrive at an optimal parameter setting. Many optimization techniques have evolved over the years to optimize the parameters by solving multiple objective functions. One such technique is principal component analysis-based grey technique.

In PCA, new uncorrelated variables are formed through the linear composition of the existing variables. Maximum number of new variables is equal to the number of existing variables. In the current work, PCA-based GRA which optimizes multiple responses by taking care of the possible correlations between the responses is applied to experimental data of the PMEDM process. The main advantage of PCA lies in the fact that once the patterns in the data have been identified, the data can be compressed without much loss of information.

GRA is based on grey relation theory in which only partial information is available. Complicated inter relationships among the responses are established through GRA. It is a normalization evaluation technique which is extended to solve complicated multi-response optimization effectively.

The optimization procedure using Grey-PCA is explained below.

Step 1. Calculate the S/N ratios for each response.

Step 2. Conduct PCA on S/N ratios corresponding to each trail to obtain uncorrelated principal component scores (PCS), by using the following equation

$$\text{PCS}_{il} = a_{i1}\eta_{1l} + a_{i2}\eta_{2l} + \ldots\ldots.. + a_{ip}\eta_{pl} \tag{30}$$

The coefficients of the lth component, i.e., a_{i1}, a_{i2}, …, a_{ip} are the elements of the eigenvector corresponding to the lth eigenvalue of the correlation matrix of the response variables.

Step 3. Since a larger PCS is always desired, normalized the principal component scores by using Equation:

$$X_{il} = \frac{\text{PCS}_{il} - \text{PCS}_l^{\min}}{\text{PCS}_l^{\max} - \text{PCS}_l^{\min}} \tag{31}$$

Step 4. Based on normalized principal component scores, calculate the grey relational coefficient (γ_{il}).

$$Y_{il} = \frac{\Delta_l^{\min} + \varepsilon^*\Delta_l^{\max}}{\Delta_{il} - \varepsilon^*\Delta_l^{\max}} \tag{32}$$

where $\Delta_{il} = |1 - X_{il}|$, ε is the distinguishing coefficient which range varies from [0, 1].

Step 5. The overall quality performance index (OQPI) values can be obtained using the following expression:

$$\text{OQPI}_i = \sum_{l=1}^{p} W_l Y_{il}, \quad \text{Where } \sum_{l=1}^{p} W_l = 1 \tag{33}$$

Step 6: Use geometric average to calculate the factor effects based on OQPI values, and then decide the optimal factor-level combination based on the-higher-the-better factor effects.

Main effect plots for OQPI values were drawn after performing all the optimization calculations as per the step wise procedure discussed earlier. From Fig. 3, the level of each individual parameter at which maximum OQPI is obtained, is considered for optimal parameter setting. Hence, for the responses considered, $C_p = 4$ g/l, $I_p = 3$ A, $T_{on} = 150$ μs and $T_{au} = 85\%$ gives the optimal parameter setting.

5. Conclusions

Machining the MMC by suspending conductive powder particle in the dielectric has shown improvement in productivity as well as surface quality. In the current work, a combination of dimensional and non-linear regression analysis is used to model material

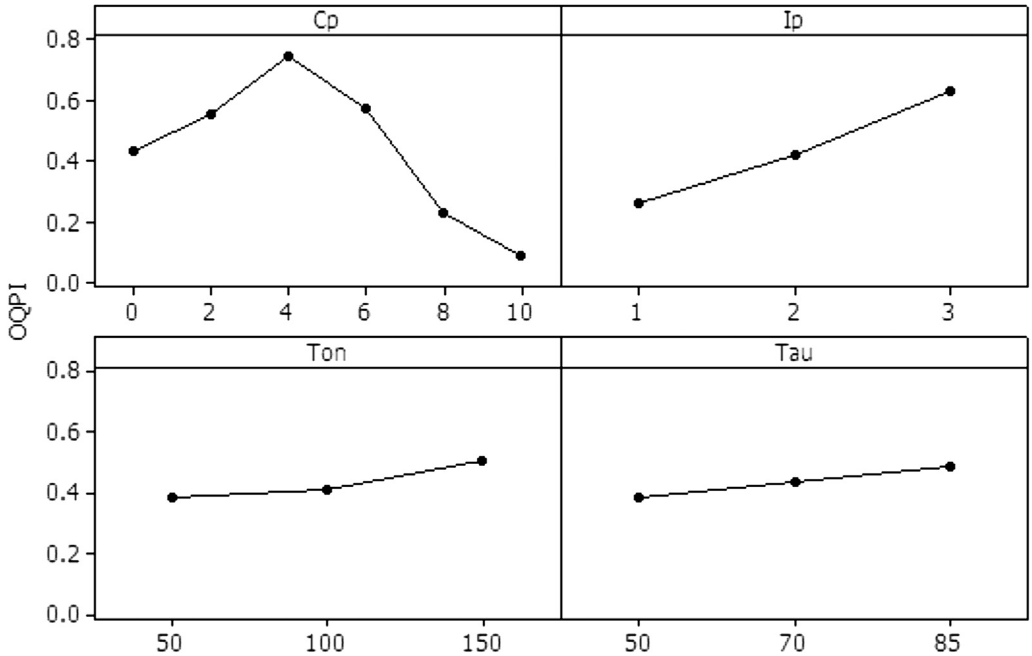


Fig. 3. Main effect plots for OQPI values.

removal rate (MRR) and surface roughness (R_a) by machining Al/Al_2O_3 MMC in aluminum suspended dielectric. Multi-objective optimization of these responses is performed using PCA-based grey relational analysis approach. The results can be summarized as follows.

- The proposed PMEDM process, using aluminum suspended kerosene dielectric for the machining of resulted in better MRR when compared to conventional EDM process. A significant decrease in R_a is also observed.
- Semi empirical models have been established for the responses, using a hybrid approach of dimensional and regression analysis in PMEDM, where the functional relationship among the process parameters and material properties is usually ambiguous.
- From the model equations, it is understood that along with machining parameters, thermal conductivity, coefficient of thermal expansion and density of the material also significantly affect both MRR and R_a. The models can be used for the further analysis of the process within the experimental range.
- PCA technique has been used to determine the weightages for responses while GRA has been used to combine the multiple objectives into single. This optimization helps to determine the suitable machining parameters for high MRR and low R_a. The recommended process parameter setting for the proposed process has been found to be $C_p = 4$ g/l, $I_p = 3$ A, $T_{on} = 150$ µs and $T_{au} = 85\%$.

Therefore, the outcome of the current research work is expected to be of significant help for the industries which are involved in processing of MMCs using EDM process. Surface integrity being one of the critical aspects in EDM, can be investigated in future during PMEDM of MMCs.

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